

"Artificial Intelligence + Traditional Culture" Empowering Applied Advanced Mathematics Classroom—A Case Study of Teaching "Monotonicity and Extrema of Functions"

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ABSTRACT

Aiming at the pain points in the teaching of "Monotonicity and Extrema of Functions" in Applied Advanced Mathematics, such as "difficulty in understanding abstract concepts, difficulty in breaking through key and difficult points, and low student participation", this paper proposes an integrated teaching model of "Artificial Intelligence + Traditional Culture". Taking GeoGebra software as the core AI tool, it visually demonstrates the correlation between the positive/negative nature of derivatives and the monotonicity of functions; and integrates the poetic artistic conception of Su Shi's "Inscription on the Wall of West Forest Temple" and the image of Lushan Mountain's terrain to materialize the abstract concept of extrema. Each peak, though not necessarily the highest across the entire range, stands as the local maximum in its vicinity; similarly, each valley, while not the lowest overall, serves as the local minimum nearby. This paper constructs a teaching closed loop of "artistic conception introduction—visual exploration—precision breakthrough—personalized consolidation". Practice shows that this model can effectively reduce students' cognitive load, improve classroom interaction and teaching effect, and provide a replicable teaching reform plan for Applied Advanced Mathematics teachers.

KEYWORDS

Artificial intelligence; Traditional culture; Applied advanced mathematics; Monotonicity and extrema of functions; GeoGebra

1 Introduction

Higher mathematics serves as a fundamental professional course, tasked with cultivating students' logical reasoning and mathematical application abilities. Within the calculus curriculum, "Monotonicity and Extrema of Functions" constitutes a core topic. It provides the foundational theory for subsequent derivative applications and offers essential mathematical support for professional skills such as engineering computation and data analysis. However, current pedagogical approaches face two significant challenges. Firstly, the abstract nature of these concepts—such as how the sign of the derivative relates to a function's increasing or decreasing behavior, or what exactly constitutes a "local optimum"—can be challenging to teach through traditional chalkboard methods, as it is hard to make these ideas tangible. Secondly, the teaching model often remains one-way, leading to passive knowledge reception among students and a consequent lack of motivation and interest for active exploration.

The integration of modern technology and traditional culture presents a viable path to address these challenges. Artificial intelligence tools, such as GeoGebra and intelligent learning platforms, demonstrate significant empowering value^[1]. They enable the visualization of abstract concepts, introduce precision into teaching processes, and allow for the personalization of learning paths, thereby directly countering the lack of intuitiveness in traditional instruction. Concurrently, traditional cultural elements can act as a conceptual bridge. Chinese classical poetry and natural images contain rich mathematical ideas. The artistic conception of "Looking horizontally, it is a range; viewed sideways, a peak" in Su Shi's "Inscription on the Wall of West Forest Temple" can establish a concrete connection between the "peak points" and "valley points" of Lushan Mountain and the extrema of functions, reducing the understanding threshold of abstract concepts^[2]. This integrated approach aligns well with the cognitive characteristics of higher vocational students, who often exhibit a stronger inclination towards a learning trajectory of "concretization, practical operation, and application." The "Artificial Intelligence + Traditional Culture" model, therefore, conforms to their learning patterns and holds promise for enhancing participation and understanding.

Given this context, the present study aims to construct an integrated teaching plan, using "Monotonicity and Extrema of Functions" as a case study. This plan is supported by artificial intelligence tools, employs traditional culture as a contextual carrier, and focuses on resolving key and difficult teaching points. The specific research objectives are threefold: first, to transform abstract mathematical concepts into intuitive visual representations and relatable cultural imagery; second, to formulate a coherent and complete teaching process structured around the phases of "introduction, exploration, breakthrough, and consolidation"; third, to provide operable teaching cases and practical tool-application guidelines for higher vocational mathematics instructors^[3]. Ultimately, this work seeks to contribute to the reform and innovation of advanced mathematics pedagogy, fostering a shift from mere knowledge transmission towards a more engaging, culturally-informed, and competency-focused classroom experience.

2 Theoretical Basis of Integrated Teaching

2.1 Constructivist Learning Theory

Constructivism holds that learning is a process in which students actively construct knowledge rather than passively accept information. The visual exploration environment provided by AI tools such as GeoGebra and the concrete carrier provided by traditional culture can help students actively construct their cognition of the concepts of monotonicity and extrema through the pathway of "observation—association—verification—summary", forming a personalized knowledge system.

2.2 Situational Teaching Theory

Situational teaching emphasizes stimulating students' learning interest and exploration desire by creating specific situations. In integrated teaching, the combination of the natural situation of "Lushan Mountain's terrain", the interactive situation of GeoGebra, and the cultural situation of poetry constructs a multi-dimensional teaching situation, allowing students to understand the essence of mathematical concepts in the situation.

3 Design of Integrated Teaching Plan

3.1 Teaching Objectives

3.1.1 Knowledge Objectives

Master the derivative criterion theorem for the monotonicity of functions; understand the definition and judgment methods of extrema; distinguish between stationary points and extremum points.

3.1.2 Ability Objectives

Be able to analyze the monotonicity and extrema of functions with GeoGebra; solve the monotonicity problems of parameter-containing functions with the help of AI tools; develop the ability to apply mathematical knowledge to solve simple practical problems.

3.1.3 Literacy Objectives

Perceive the internal connection between traditional culture and mathematics; cultivate logical thinking and innovative awareness; improve the literacy of AI tool application and cultural confidence.

3.2 Key and Difficult Points in Teaching

Key Points: Application of the derivative criterion for determining function monotonicity; the procedures for solving function extrema.

Difficult Points: Analysis of monotonicity for functions containing parameters; understanding the "local optimum" nature of extrema; differentiating between stationary points and extremum points.

3.3 Core Tools and Resources

- (1) AI Tools: GeoGebra (for function visualization), ChaoXing Learning Platform (intelligent teaching platform).
- (2) Cultural Resources: The original text of Su Shi's poem *On the Wall of West Forest Temple (Ti Xilin Bi)*; images and dynamic videos of the terrain of Lushan Mountain.
- (3) Teaching Resources: personalized practice exercises, error diagnostic system.

3.4 Teaching Implementation Process (40 minutes)

Artistic Conception Introduction: Bridging with Traditional Culture to Stimulate Inquiry Interest (8 minutes)

Cultural Situation Creation: In the teaching introduction session, play a dynamic video showing the rolling terrain of Lushan Mountain, present the poetic conception of the lines "From the side, it is a range; from the end, a peak; Far and near, high and low, it never looks the same" from Su Shi's poem *On the Wall of West Forest Temple*, and then ask students: "What are the core characteristics of Lushan Mountain's terrain? What kind of features do the 'peak points' and 'valley points' exhibit within local areas?"^[4].

Mathematical Association Guidance: Guide students to discover that "peak points" are the highest points in the surrounding area and "valley points" are the lowest points nearby, thus naturally leading to the embryonic form of the concept of "extrema". Then, draw an analogy between "the undulating changes of mountain terrain" and "the increasing and decreasing changes of function values" to introduce the theme of this lesson—Monotonicity and Extrema of Functions.

Design Intention: With traditional culture and natural imagery as the bridge, and with the assistance of the AI tool GeoGebra, visualize abstract mathematical concepts, thereby stimulating students' inquiry interest and cultural resonance [5].

Visualized Inquiry: Empowering with AI Tools to Understand Core Concepts(15 minutes)

Session 1: Visualization of Function Monotonicity (8 minutes)

Teacher's Operation: Open GeoGebra, input the basic function $y = x^3 - 3x$, use the derivative command to generate $y' = 3x^2 - 3$, and apply the "IntegralBetween" command to mark the intervals where $y' > 0$ ($x < -1$ or $x > 1$) in light blue and the intervals where $y' < 0$ ($-1 < x < 1$) in light red.

Interactive Inquiry: Teachers guide students to drag the dynamic point on the function curve, observe that "the function value increases when the dynamic point moves in the light blue area and decreases when it moves in the light red area", thus enabling students to intuitively understand the correlation rule that "positive derivative $\rightarrow$ increasing function, negative derivative $\rightarrow$ decreasing function".

Extension Session: Teachers use the slider function of GeoGebra to add a parameter " k ", modify the function to $y = x^3 - 3kx$, and then guide students to drag the slider to observe the synchronous changes of derivative zeros ($\pm\sqrt{k}$) and shaded intervals when " k " changes, and help students understand the monotonicity rules of parameter-containing functions, as shown in the figure below.

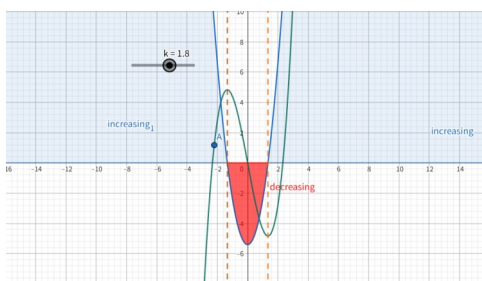


Figure 1 Visualization of Extremum Concepts (7 minutes)

Cultural Connection: Reconnect with the terrain of Lushan Mountain, and point out that "the extremum points of a function are just like the peak points and valley points of Lushan Mountain, which are the maximum and minimum values in local ranges".

Tool Demonstration: Mark the extremum points $(-1, 2)$ and $(1, -2)$ of the function $y = x^3 - 3x$ in GeoGebra, use the trace function to show the changing trend of function values near the extremum points, and emphasize that "extrema are local properties rather than global maximum and minimum values".

Concept Differentiation: Generate the functions $y = x^2$ and $y = x^3$, compare their derivative conditions and function shapes at $x = 0$, and clarify the conclusion that "stationary points are not necessarily extremum points".

Precise Breakthrough Session (12 minutes)

Differentiation between Stationary Points and Extremum Points

Error Analysis System: Present typical wrong answers from students, and use AI tools to label the causes of errors (e.g., confusing the definitions of stationary points and extremum points, failing to verify the sign changes of derivatives).

Interactive Differentiation: Organize students to use GeoGebra to plot three types of function graphs—functions with stationary points that are not extremum points, functions with extremum points that are not stationary points, and functions with points that are both stationary and extremum points—to deepen their understanding of the concepts.

Personalized Consolidation Session (5 minutes)

Tiered Assignments: Based on students' in-class response data, the Chaoxing Study Online platform pushes tiered homework assignments:

- (1) Basic Level: Solving monotonicity and extremum problems of elementary functions;
- (2) Intermediate Level: Comprehensive application of parameter-containing functions;
- (3) Advanced Level: Analyzing extremum in practical problems (e.g., *profit maximization*) with AI tools.

Extended Task: Ask students to independently design a complex function, analyze its monotonicity and extrema with GeoGebra, generate a visual analysis report, and submit it to the course platform for communication and sharing.

4 Teaching Effectiveness and Reflection

4.1 Teaching Effectiveness

4.1.1 Student Perspective

Based on classroom observation and post-course surveys, over 90% of students found the "Traditional Culture + AI" model made abstract concepts easier to understand. 85% of students were able to independently use GeoGebra to analyze function monotonicity and extrema. Classroom engagement increased by approximately 30% compared to traditional instruction.

4.1.2 Teacher Perspective

The visualization and automation features of AI tools reduced the teacher's instructional workload, allowing them to focus more on explaining key and difficult points as well as guiding students. Dynamic data feedback enabled the teacher to accurately assess student comprehension, facilitating a shift towards "teaching informed by learning."

4.2 Teaching Reflection

4.2.1 Strengths

The integration of traditional culture enhanced the instructional appeal and cultural depth. The application of AI tools achieved intuitive and precise teaching. Their combination effectively addressed key pain points in higher vocational applied advanced mathematics teaching.

4.2.2 Considerations

Over-reliance on AI tools must be avoided, with emphasis maintained on students' understanding of core concepts and logical reasoning. The integration of traditional culture should be natural and targeted, avoiding forced connections. For students less proficient with AI tools, simplified operation guides and peer support within small groups should be provided.

5 Conclusion

The integrated teaching model of "Artificial Intelligence + Traditional Culture" provides an innovative pathway for the instruction of "Monotonicity and Extrema of Functions" in higher vocational education. By leveraging the visualization capabilities of AI tools such as GeoGebra, it addresses the difficulty students have in grasping abstract concepts. The cultural imagery from Su Shi's poem "Inscribed on the Wall of West Forest Temple" and the topography of Mount Lu serves as a bridge between concrete imagery and abstract mathematical thought. Furthermore, the diversified application of intelligent teaching platforms facilitates precise and personalized instruction.

This model not only effectively enhances the teaching and learning outcomes for "Monotonicity and Extrema of Functions" but also offers a valuable reference for reforming the pedagogy of other chapters in higher vocational mathematics. It contributes to the transformation of the advanced mathematics classroom from a model focused primarily on "knowledge transmission" to one that emphasizes "capability cultivation" and "cultural immersion," thereby elevating the quality and substance of mathematics education in higher vocational institutions.

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